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PLANNING AND CONTROL UNDER RISK

Final Report

William S. Jewell

November 1977

U. S. Army Research Office

DAAG29-77-G-0040

Operations Research Center
University of California, Berkeley

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ABSTRACT

This is the Final Report in a twelve-year effort to model stochastic phenomena and develop decision-making techniques under risk and uncertainty. Recent research areas which received major emphasis were:

(1) Basic risk decision models, with emphasis on determining the structure of optimal policies in the face of unknown parameters in the relevant risk distributions;

(2) Data collection and parameter estimation with emphasis on linearized Bayesian methods.

I. INTRODUCTION

During the past twelve years under this contract, "Planning and Control Under Risk," (DAAG 29-77-G-0040 and predecessor contracts), faculty and students at the Operations Research Center, University of California, Berkeley, have tackled a variety of planning and control problems, with increasing emphasis on decision problems under risk; a list of publications since 1965 is in the Appendix. In this, our final report, we summarize the work that was completed during the final period of this grant, November 1, 1976–November 1977.

The motivation for this program of research was that almost all realistic decision problems in military and industrial operations contain elements of risk or uncertainty. These unknown elements limit the information we can gather about the current state of system, and make future states unknowable, except in a statistical sense, thus strongly influencing attitudes towards the decisions to be made. In other words, the utility of any given decision can only be forecast in a probabilistic manner, and sometimes cannot be evaluated retrospectively without residual uncertainty. Thus, planning and control strategies must always reflect these risks and uncertainties.

There are a variety of models and methods which appear under the heading of risk theory; generally they share the following characteristics:

(1) There is usually a probabilistic law of motion governing the system. In simple cases, this may be a binary gamble, increasing or decreasing the wealth available for future risk taking; in more complicated models, a Markov-renewal or diffusion law may change the state of the system over time.

(2) Transitions usually create costs or generate profits, and these, too, may be uncertain.

(3) There is usually a concern about process boundary conditions, which can lead to ruin (financial, insurance, gambling models), to catastrophes (design of dams and nuclear reactors, responses to fires and epidemics, etc.), or simply to the termination of the game (reliability and human- or corporate-lifetime models). Sometimes the ability to deliberately terminate a (losing) game is the feature of primary concern, as in optimal stopping rules. Conversely, in some stochastic optimization problems, there may be some mathematical embarrassment associated with continuing "forever," and turnpike theorems, discounting, or absorbing states may have to be invoked to avoid analytic difficulties.

(4) Expected total return or average rate of return may not be a satisfactory system objective; for instance, the decision-maker may be just as concerned about the fluctuations of reward under a certain policy as about the average reward. This may lead to the use of utility theory as an axiomatic way of specifying the decision-maker's risk-aversion, to more explicit multiple-objective formulations, or to nature-as-the-opponent minimax functionals. Many interesting risk-sharing problems require multiple-player pareto-optimality approaches. Problems with long horizons usually require specific recognition of the utility of time.

(5) Finally, there are always important data measurement and parameter estimation problems associated with decision under risk. Not only may the observations themselves be subject to error, but there are usually many more parameters to estimate for stochastic problems than for deterministic ones. An important part of many dynamic decision models is the provision for continuing updating of the estimators.

There may or may not be prior information upon which to base initial decisions. The mechanics of combining estimates and data are often laborious in a real problem, and the costs of updating information must often be included in setting up a model.

We feel strongly that this can continue to be a fruitful research area. In fact, a heightened sensitivity to the problems of risk within our industries and government agencies has been demonstrated in recent years, as a new problems of the environment, concerns over worker and population safety, resource and energy shortages, etc. have been encountered. Economic planning must now take into account possible sudden shifts in available resources, the uncertainty of long-term plans, and the financial interdependence of sectors of our society. We no longer have the luxury of a certain world, but must develop decision options for a variety of uncertain alternate scenarios.

In closing this long and prductive relationship with the Army Research Office, the principal investigators would like to thank Army personnel for their continued support and encouragement. The educational benefits derived from this effort were immeasurable, and it is hoped that the research output was ultimately useful.

II. AREAS OF RESEARCH (1976-1977)

A. Models of Decision Under Risk

1. Renewal Decision Models

Consider a system that must operate for a fixed length of time and suppose that a certain component is essential for the system to be operative. When this component fails it must be replaced. However there are n different types of this component that can be used; the ith type costing the amount C_i and functioning for an exponentially distributed length of time with rate λ_i. Thus, typically, the more expensive types will last longer. This model was considered in [1] where the main problem was to determine the optimal strategy for assigning replacements for the failed components as a function of the remaining life time of the system. It was shown that the optimal policy has a particularly nice structure; namely that the time axis can be divided into n intervals such that when a replacement has to be made it is optimal to select a spare from the category having the ith largest value of λC whenever the remaining time falls into the ith closest interval to the origin. This special structure was then exploited to obtain an efficient algorithm for the determination of the critical switch points.

Other variations of the above model were also considered in [1]. In particular the case of only 2 types of components is considered when there is only a single spare of one of these types and an infinite surplus of the other. Also the optimal policy is completely determined when there is only a finite number of certain of the types under the assumption that if rebate is given for the component in use at the end of the systems life.

2. Gambling Models

In order to obtain some insight into the structure of optimal policies in risk models, a class of gambling models, useful as simple prototypes for risk models, was considered in [2]. For a variety of objectives, it was shown that if the game is favorable to the player, then he should play as timidly as possible; that is, always make the smallest bet. A model in which the gambler is also given the option of working is considered, and it is shown that if the available gambles are unfavorable then the strategy which minimizes the gambler's expected time to reach some preassigned goal is the strategy that always calls for working. For the same model it is also shown that if the work option is only available at certain times (namely, when the gambler is broke) then the optimal

gambling strategy is to play boldly. These results were obtained by developing some new general results in dynamic programming, also given in [2].

The study of these models was continued in the Ph.D. thesis [3] of E. Subelman, a student of Professor Ross. This thesis considered the problem where a decision-maker is allowed to gamble with his objective being to maximize the (expected) utility of his final fortune. It is supposed that if he bets an amount y then his return from this gamble will be yR, where R is a random variable whose distribution is known to the gambler, the classical case being when

$$R = \begin{cases} 2 & \text{with probability } p \\ 0 & \text{with probability } 1-p \,. \end{cases}$$

The objective of this research is to determine properties of $y_n(x)$, the optimal bet when the gambler's fortune is x and there are n gambles to go, under the assumption of an (arbitrary) concave utility function. Some of the results obtained were that, in the classical case, the optimal amount to save and to strive for are both increasing in the gambler's fortune. That is, in the classical case, both

$$x - y_n(x)$$

and

$$x + y_n(x)$$

are increasing functions of x. In addition it is shown that, as a function of p, $y_n(x)$ is nondecreasing. Thus the more favorable the game the more one should bet. These results were then generalized to the adaptive situation in which p is unknown and information about p is thus obtained from the outcome of the gambles.

3. Optimal Inspection Policies

In a Ph.D. thesis written under Ross' supervision and partially supported by ARO, Levin [4] considered a wide variety of models for maintaining systems when inspection is costly. In particular, his models include the classical assumption that a system changes states in accordance with a Markov chain, but the true state remains unknown until a costly inspection is made. Thus one has to balance the cost of inspection with the possibly high costs of being in a bad state. Levin has obtained some new and interesting structural results for this model. Another model considered is an inventory situation in which, for a given order, the number of items actually received is a random variable; once again interesting structural results are obtained.

4. The Newsboy Problem

One of the most elementary risk decision problems is the no-carryover ordering problem known as the newsboy problem. As a preliminary to studying a Bayesian

version of this problem (see Section II), the following variant was investigated by R. Levin, under Jewell's supervision [5]:

> Given a known demand distrubution, $F(x)$, a "newsboy" places an order z_1. If then actual sales, $\tilde{x}$, grow larger than z_1, he can make an instantaneous replenishment in amount z_2, and then if $\tilde{x} > z_1 + z_2$, he can make another instantaneous replenishment, and so on ..., to some given maximal number of reorders, N. Costs include not only variable costs or ordering and profits per sale, but possibly fixed costs of replenishment.

Naturally, the initial orders depend not only upon the cost parameter, but upon the number of remaining possible orders (one does not always reorder even when permitted), as well as the shape of the residual demand distribution.

5. <u>Other Work Completed</u>

In November, 1976, W. S. Jewell was invited to present a paper to the Royal Society (London) Discussion Meeting on "The Use of Operational Research and System Analysis in Decision Making." Preparation of survey paper, "The Analytic Methods of Operations Research" [6] was partly supported by this contract, and by the Office of Naval Research.

B. Estimation Problems

1. Introduction

Most multistage decision problems under risk require processing of large amounts of data, either for initial estimates of the parameters, or to provide continuing updating as new information is received from prior decisions. Thus, efficient methods of estimation are important.

Suppose we can observe a random variable $\tilde{x}$ which depends upon a parameter θ in a known way, via the *likelihood density* $p(x|\theta)$; we assume a *prior* parameter density, $p(\theta)$, is available. Prior-to-data predictions about an average $\tilde{x}$ can then be made through the mixed density $p(x) = E_\theta p(x|\theta)$.
Now suppose that, as a result of some decision about experimentation and sampling, we observe n independent samples of the random variable, $\underline{x} = \{\tilde{x}_t = x_t \; ; \; t = 1,2, \ldots, n\}$. By using Bayes' theorem, the parameter density posterior-to-data becomes

$$(1) \qquad p(\theta|\underline{x}) = \frac{\Pi p(x_t|\theta)p(\theta)}{\int \Pi p(x_t|\psi)p(\psi)d\psi} \; .$$

In decision problems, we are not usually so interested in estimating the true value of parameter θ , as we are in the *forecast density* for $\tilde{x}_{n+1} = y$, the next observation. Thus, practical control problems require the calculation of:

$$(2) \qquad p(y|\underline{x}) = E_{\theta|\underline{x}} p(y|\theta) = \int p(y|\theta)p(\theta|\underline{x})d\theta$$

or its moments. For instance, in decision analysis, as decisions

to perform experiments are made, the information from the experiments is used to recompute expected utilities of one or more future actions.

Practically speaking, computations via (1) and (2) are laborious, and require either large computer capability, or the use of a few well-known *natural conjugate prior families* of likelihood and prior. This requires knowing (or assuming) a great deal of structure information about $p(\theta)$ and $p(x|\theta)$. But, as indicated above, most of this information is "wasted," especially if we only want moments or an expected utility.

In the 1920's American actuaries developed a class of estimators called "credibility formulae" to predict the *mean* of the next risk outcome, $\tilde{x}_{n+1}$, given the "experience data," $\underline{x}$. Using simple heuristic arguments, they derived a linear forecast formula $f(\underline{x})$:

$$
\begin{aligned}
E\{\tilde{x}_{n+1}|\underline{x}\} \approx f(\underline{x}) &= (1 = z)\cdot m + z\left(\frac{1}{n}\sum_{t=1}^{n} x_t\right)\cdot \\
z &= \frac{n}{n + N}
\end{aligned}
\tag{3}
$$

where m is the prior mean of $p(y)$ (no data), $\frac{1}{n}\sum x_t$ is the sample mean of data, and N is a time constant, chosen originally in an ad hoc manner. The most interesting features of this formula are: (1) It uses the data in a simple linear fashion, mixing it with the prior estimate; (2) it worked extremely well for 40 years in casualty insurance experience rating.

In the 1950's it was discovered that (3) was in fact the *exact* Bayesian result $E\{\tilde{x}_{n+1}|\underline{x}\}$ (gotten from (1) and (2)) if the prior and likelihood were the natural conjugate priors: Beta-Binominal, Gamma-Poisson, and Normal-Normal. Then in 1967, it was shown that the credibility forecast (3) is the best *least-squares approximation* for arbitrary distributions, if N is chosen as the ratio of the two components of prior variance.

2. Recent Research

In an extensive research effort supported by ARO since 1973, we have been able to develop a variety of interesting and useful extensions to credibility estimators:

(1) Proof that (3) is exact Bayesian for the Koopman-Pitman-Darmois exponential family for which $\bar{x}$ is a sufficient statistic;

(2) Extensions of least-square and exact Bayesian theory to vector valued random variables;

(3) Estimates of probabilities, $P(y|\underline{x})$, for fixed y;

(4) Numerous extensions to special model structures, including hierarchial models;

(5) Direct and inverse Bayesian regression estimators, with applications to calibration of instruments and measurements of network flows.

These developments (References 8 through 25) have been reported in detail in previous proposals and reports.

During the interval from July 1976 to November 1976, related but independent research was carried out for the Lawrence Livermore

Laboratory on estimation problems associated with material accountability problems [24] [25].

In August, 1976, W. S. Jewell was invited to Osaka, Japan to the annual Management Science Colloquium of Japan. For this occasion, he prepared a survey paper on the many recent contributions to credibility theory [23].

In [21], (supported by ARO under a different contract), a new family of likelihoods, called the *proportional-hazard-family*, was proposed to facilitate Bayesian computations in life-testing, where there are *incomplete observations* of the form $\tilde{x} > T$. This work has been further extended in [26]. One of the questions examined in this paper is the following:

> An item with random lifetime density $p(x|\theta)$ has "lived" until age T . Taking into account the "learning" about θ which occurs from the datum $\{\tilde{x} > T\}$, could we predict the distribution of remaining life in any different way than by using the prior mixed density $p(x) = \mathcal{E}p(x|\tilde{\theta})$ in the obvious way?

Not surprisingly, the single datum provides no additional information about the remaining life through changing estimates of θ . If, however, there are *several* items, all with the same parameter, that are "on test" at age T , then there is a changed perception about the common remaining life distribution. This result has implications for equipment sinking fund analyses, and other "learning reserves" models.

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November 1, 1976 - November 19, 1977

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Jan W. Kwiatkowski

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Linda Yellin

DEGREES AWARDED

November 1, 1976 - November 19, 1977

Robert D. Levin, Ph.D.
June 1977

Shun-Chen Niu, Ph.D.
December 1977